On the Feasibility of Single-Trace Attacks on the CDT Gaussian Sampler

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Motivation

Quantum computers are expected to be widely available in the next decade
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Quantum attacks pose a threat to classical cryptography
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Quantum attacks pose a threat to classical cryptography

Urgent need for quantum-resistant schemes
## Motivation

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Lattice-based key encapsulation mechanisms

- The security of lattice-based cryptography often relies on the Learning with Errors problem (LWE)

- An LWE instance contains the secret vectors blinded with a noise vector (error)

- Usually, the noise vectors are taken from a Gaussian distribution, typically acquiring many samples for a single run of the scheme
Key encapsulation mechanisms

Figure: Simplified Example of TLS Connection Establishment
FrodoKEM: Decapsulation (simplified)

**Algorithm 1** \text{FrodoKEM.Decaps}(c_1||c_2 \text{ and } sk(s||S))

1: \(B, B', C \leftarrow \text{Frodo.Unpack}(c_1, c_2, b)\)
2: Compute \(M \leftarrow C - B'S\)
3: Compute \(\mu' \leftarrow \text{Frodo.Decode}(M)\)
4: Sample error matrix \(S', E', E''\)
5: Compute \(B'' \leftarrow S'A + E'\) (\(A\) is public)
6: Compute \(V \leftarrow S'B + E''\)
7: Compute \(C' \leftarrow V + \text{Frodo.Encode}(\mu')\)
8: \textbf{if} \(B'||C = B''||C'\) \textbf{then}
9: \hspace{1em} return \((c_1||c_2||\text{SHAKE}(c_1||c_2||\mu'))\)
10: \textbf{else}
11: \hspace{1em} return \((c_1||c_2||\text{SHAKE}(c_1||c_2||s))\)
12: \textbf{end if}
FrodoKEM: Decapsulation (simplified)

Algorithm 1 FrodoKEM.Decaps( $c_1\|c_2$ and $sk(s\|S)$)

1: $B, B', C \leftarrow$ Frodo.Unpack($c_1, c_2, b$)
2: Compute $M \leftarrow C - B'S$
3: Compute $\mu' \leftarrow$ Frodo.Decode($M$)
4: Sample error matrix $S', E'$, and $E''$
5: Compute $B'' \leftarrow S'A + E'$ ($A$ is public)
6: Compute $V \leftarrow S'B + E''$
7: Compute $C' \leftarrow V +$ Frodo.Encode($\mu'$)
8: if $B'\|C = B''\|C'$ then
9: return $(c_1\|c_2\|SHAKE(c_1\|c_2\|\mu'))$
10: else
11: return $(c_1\|c_2\|SHAKE(c_1\|c_2\|s))$
12: end if
The CDT Gaussian sampler

**Algorithm 2** Constant-time CDT sampling

**Require:** CDT $\psi$ of length $l, \sigma, \tau$

**Ensure:** Sampled value $S$

1. $S \leftarrow 0$
2. $\text{rnd} \leftarrow [0, \tau \sigma) \cup \mathbb{Z}$ uniformly at random
3. $\text{sgn} \leftarrow [0,1] \cup \mathbb{Z}$ uniformly at random
4. for $(i = 0 ; i < l - 1; i + +)$ do
5.   $S \leftarrow (\psi[i] - \text{rnd}) >> 15$
6. end for
7. $S \leftarrow ((-\text{sgn}) \wedge S) + \text{sgn}$
8. return $S$

- The Gaussian sampler is based on a cumulative distribution table CDT
- The CDT length depends on deviation of the Gaussian distribution $\sigma$ and the Tailcut $\tau$
- The implementation is constant-time
- A sign bit is assigned to the positive output sample
Side-channel analysis of the CDT Gaussian sampler

Figure: Overlapped power consumption measurement during the execution of the CDT sampler on an 8-bit Harvard board equipped with an XMega micro-controller; the red color corresponds to the sampling of the value 1, while the blue color corresponds to the sampling of the value 0
Experiments vs. Real-world scenario

- 8-bit Harvard board are used in literature
- An X-Mega micro-controller which is especially common in educational embedded applications
- In contrast, Cortex-M boards have been embedded in tens of billions of consumer devices
- The frequency and/or the sampling rate have dramatic effect on the accuracy of the power consumption traces
- Noise filtering tools
Measurements on Cortex-M4

Figure: Overlapped Power consumption measurement during the execution of the CDT sampler on a Cortex-M4 equipped with an STM32F4 microcontroller; the red color corresponds to the sample of the value 0, while the blue color corresponds to the sampling of the value 1
Measurements with different frequencies

Execution of the Gaussian sampler on 32-bit STM32F4 Microcontroller at different frequencies

**Figure:** Overlapped Power consumption measurement during the execution of the CDT sampler on a 32-bit Cortex board equipped with an STM32F4 microcontroller with different frequencies.
Power consumption measurement

We write the power consumption at a specific point of time as the following:

\[ P = P_{\text{op}} + P_{\text{data}} + P_{\text{noise}} + P_{\text{const}} \]
Why machine-learning side channel analysis?

- No assumptions on the introduced noise
- Automated selection of Points of Interest (POI)
- Efficient management of large traces/small profiling sets
- Resilience against the addition of useless (i.e. non-informative) leakage samples in the traces
Threat models in profiling attacks

- The classical threat model: A single-device-model

- Portability threat model: A cross-device attack (identical devices, homogeneous devices, heterogeneous devices, etc.)

- Non-profiling supervised threat model: A differential deep learning analysis
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Machine-learning profiling attack on FrodoKEM: Profiling phase
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- The list of these noisy measurements is split into training, validation, and a test set of the Multi-Layer Perceptron (MLP) machine-learning classifier
- The attacker should train a classifier for each board
- Tuning the hyper-parameters of our machine-learning model is of particular importance because it influences the accuracy
- We captured 20,000 power consumption traces. We set 18,000 of them for training and testing and 2,000 for validation.
Machine-learning profiling attack on FrodoKEM: Attack phase

Victim's device → Power trace unknown samples → Trained neural network

capture → predict → samples guess
Algorithm 1 FrodoKEM.Decaps (c₁∥c₂ and sk(s∥S))

1: \( B, B', C \leftarrow \text{Frodo.Unpack}(c_1, c_2, b) \)
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3: \( \text{Compute } \mu' \leftarrow \text{Frodo.Decode}(M) \)
4: \( \text{Sample error matrix } S', E', \text{ and } E'' \)
5: \( \text{Compute } B'' \leftarrow S'A + E' \text{ (A is public)} \)
6: \( \text{Compute } V \leftarrow S'B + E'' \)
7: \( \text{Compute } C' \leftarrow V + \text{Frodo.Encode}(\mu') \)
8: \( \text{if } B'∥C = B''∥C' \text{ then} \)
9: \( \quad \text{return } (c_1∥c_2∥\text{SHAKE}(c_1∥c_2∥\mu')) \)
10: \( \text{else} \)
11: \( \quad \text{return } (c_1∥c_2∥\text{SHAKE}(c_1∥c_2∥s)) \)
12: \( \text{end if} \)
Session key recovery

Having the values of $S'$ and $E''$, the attacker computes the matrix $V$:

$$V = S'B + E''$$
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Then, the attacker obtains the matrix $\text{Encode}(\mu')$:

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Hence, $\mu'$ can be written as:

$$\mu' = \text{Frodo.Decode}(C' - S'B - E'')$$
Conclusion

• We investigated the feasibility of single trace attacks against the CDT Gaussian sampler
• We proved that in real-world circumstances the accuracy of the attack decreases
• We present a machine-learning classifier leveraging the accuracy of the attack to 100%
• We apply our attack on FrodoKEM in real-world circumstances and present a proof of concept of our attack implementation
Thank you for your attention