Removing the Field Size Loss from Duc et al.'s Conjectured Bound for Masked Encodings

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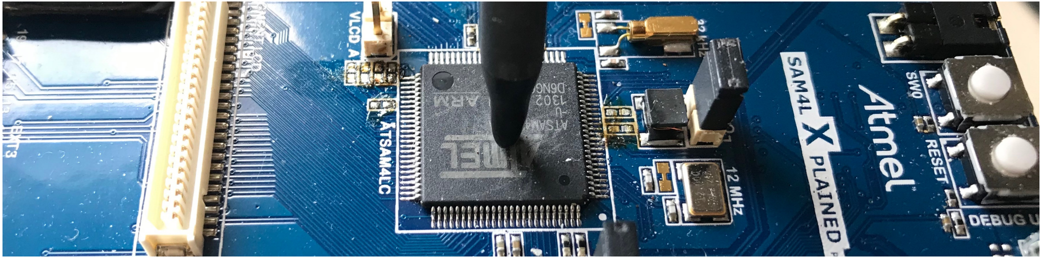
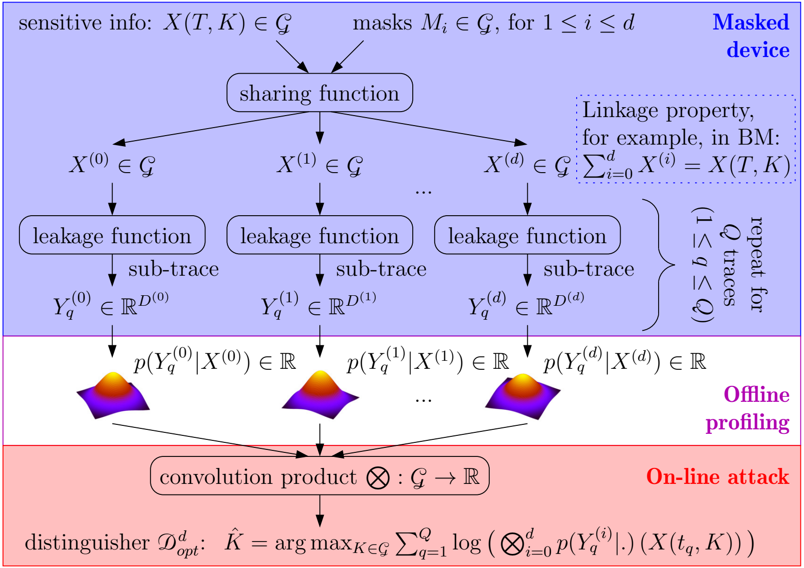
Side Channel

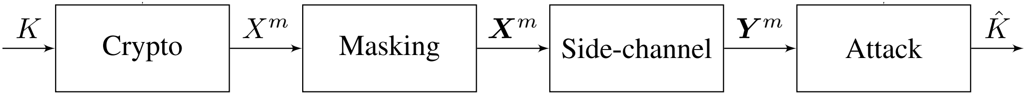


Masking as a Countermeasure

Example of Boolean Masking (BM) in $\mathcal{G} = \mathbb{Z}_{2^n}$

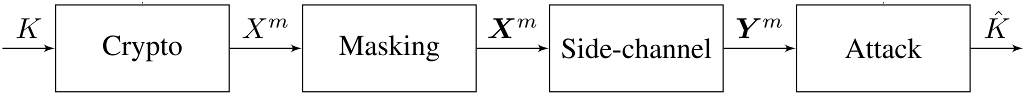


Theoretical Problem



- compute sensitive values $X \sim \mathcal{U}(M)$ in an Abelian group $(\mathcal{G}, \oplus)$ of order $M = |\mathcal{G}|$, which depends on some secret K;
- secret sharing computation: X is split into $d+1$ random shares $X_i \sim \mathcal{U}(M)$: $X = X_0 \oplus X_1 \oplus \cdots \oplus X_d$ in $\mathcal{G}$;
- this is a dth-order masking countermeasure against noisy leakages $Y_0, \ldots, Y_d$;
- defender's (worst case) problem: Evaluate the *minimum number of measurements* m that can achieve the *best possible performance* (SR), i.e., probability of success $\mathbb{P}_s = \mathbb{P}_s(K|\mathbf{Y}^m)$ given by the MAP rule.

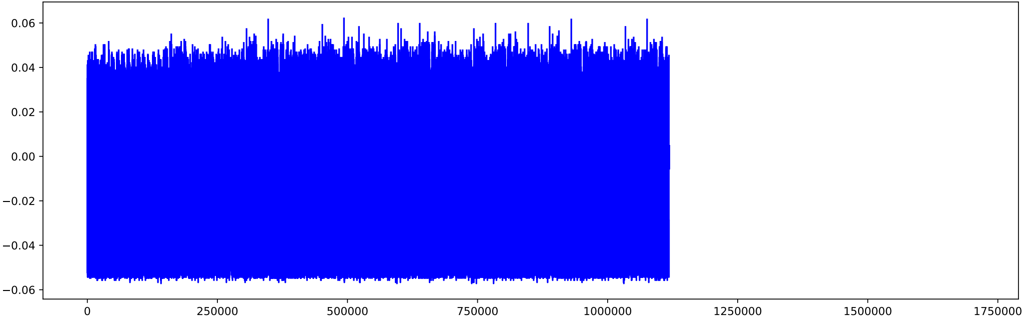
Theoretical Problem



- compute sensitive values $X \sim \mathcal{U}(M)$ in an Abelian group $(\mathcal{G}, \oplus)$ of order $M = |\mathcal{G}|$, which depends on some secret K;
- secret sharing computation: X is split into $d+1$ random shares $X_i \sim \mathcal{U}(M)$: $X = X_0 \oplus X_1 \oplus \cdots \oplus X_d$ in $\mathcal{G}$;
- the adversary performs m measurements to achieve a given success rate (SR);
- defender's (worst case) problem: Evaluate the *minimum number of measurements* m that can achieve the *best possible performance* (SR), i.e., probability of success $\mathbb{P}_s = \mathbb{P}_s(K|\mathbf{Y}^m)$ given by the MAP rule.

Evaluation Context: High Dimension Traces

A Trace Sample from the DPA Contest



Duc+*al* Evaluation Bound

"Making Masking Security Proofs Concrete," Duc, Faust & Standaert, Eurocrypt2015

Theorem (Duc+al evaluation bound)

$$m \geqslant \frac{\log \frac{1-1/M}{1-\mathbb{P}_s}}{-\log(1-(\frac{M}{\sqrt{2\log e}})^{d+1}\prod_{i=0}^{d}\sqrt{I(X_i;Y_i)})} \quad (1)$$

For high noise, the denominator is $\approx (\frac{M}{\sqrt{2\log e}})^{d+1} \prod_{i=0}^{d} I(X_i;Y_i)^{1/2}$ which is too large even for moderate SNR.

Duc+*al* Long Standing Conjecture

"Making Masking Security Proofs Concrete," Duc, Faust & Standaert, Eurocrypt2015

Conjecture (Duc+*al*, revisited)

$$m \geqslant f(SR)\left(\prod_{i=0}^{d} \frac{I(X_i;Y_i)}{\tau}\right)^{-\gamma}$$

where

- *f is a function independent (or mildly depending) on the field size M;*
- *$\tau = 1$[1] is a noise amplification threshold;*
- *$\gamma = 1$ is the exponent yielding an effective masking order $d' = \gamma d$.*

[1] When the mutual information is expressed in bits.

Masure+*al* Evaluation Bound

"A Nearly Tight Proof of Duc et al.'s Conjectured Security Bound for Masked Implementations," Masure, Rioul & Standaert CARDIS2022

Theorem (Masure+*al*)

$$m \geqslant \frac{d(\mathbb{P}_s || \frac{1}{M})}{\log(1 + \frac{M}{2} \prod_{i=0}^{d} \frac{2}{\log e} I(X_i; Y_i))}$$

- independently, "On the Success Rate of Side-Channel Attacks on Masked Implementations," Ito, Ueno & Homma, CCS2022 derived the same expression with $M-1$ instead of $M/2$. Their proof uses Pinsker inequality and the Fourier transform on $\mathcal{G} = \mathbb{Z}_2^n$ (Parseval)

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- for high noise, the denominator is $\approx M(\frac{2}{\log e})^d \prod_{i=0}^{d} I(X_i; Y_i)$ which improves upon $(\frac{M}{\sqrt{2\log e}})^{d+1} \prod_{i=0}^{d} I(X_i; Y_i)^{1/2}$. Yet it still gives loose security guarantees compared to actual attacks (factor 256 for AES)

Revisiting Mrs. Gerber's Lemma

"A Theorem on the Entropy of Certain Binary Sequences and Applications: Part I," Wyner & Ziv,

Transaction Information Theory, 1973

Lemma (MGL)

$h(h^{-1}(x) \star h^{-1}(y))$ is convex in x for fixed y, where $h(p) = -p\log p - (1-p)\log(1-p)$ and $p \star q = p(1-q) + (1-p)q$.

Lemma (Revisited MGL)

For $\mathcal{G} = \mathbb{Z}_2$,

$$I(X, \mathbf{Y}) \leqslant \varphi\left(\prod_{i=0}^{d} \varphi^{-1}(I(X_i, Y_i))\right)$$

where $\varphi(x) = \log(2) - h(\frac{1-x}{2})$ is the binary DFT of h.

Revisiting Mrs. Gerber's Lemma

"The EPI and MGL for Groups of Order 2n," Jog & Anantharam, **Transaction Information Theory, 2014**

Lemma (Revisited Extended MGL)

For $|\mathbb{G}| = 2^n$,

$$I(X, \mathbf{Y}) \leqslant \varphi(\prod_{i=0}^{d} \varphi^{-1}(I(X_i, Y_i)))$$

where $\varphi(x) = \log(2) - h(\frac{1-x}{2})$ *and the product is taken only over* $I(X_i; Y_i) < \log 2$.

The number $d' \leqslant d$ of shares such that $I(X_i; Y_i) < \log 2$ can be seen as the effective masking order of the implementation. For correct masking implementation and under high noise $d' = d$.

Consequence for Masking Security (Our Contribution)

With the condition that there exists at least one $I(X_i; Y_i) < \log(2)$:

Theorem (Main Theorem)

For alphabet size $M = 2^n$,

$$m \geqslant \frac{d(\mathbb{P}_s || \frac{1}{M})}{\varphi(\prod_i \varphi^{-1}(I(X_i; Y_i)))}$$

For high noise (all $I(X_i; Y_i) < \log(2)$), since $\varphi(x) \approx (\frac{\log e}{2})x^2$ as $x \to 0$, the denominator is $\approx (\frac{1}{\log e})^d \prod_{i=0}^{d} I(X_i; Y_i)$

The derived bound is optimal without further assumption.

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High Noise Regime

Theorem (Main Theorem with High Noise)

For alphabet size $M = 2^n$, and high noise

$$m \gtrsim \frac{d(\mathbb{P}_s || \frac{1}{M})}{(\frac{2}{\log e})^d \prod_{i=0}^{d} I(X_i; Y_i)}$$

Proof.

$\varphi(x) \approx (\frac{\log e}{2})x^2$ as $x \to 0$ □

This proves Duc's conjecture except for the noise threshold $\tau \approx 0.72$ and not 1. Though this is only with the Taylor expansion in zero, it seems that there is no real "noise threshold".

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Illustration for $M = 256$

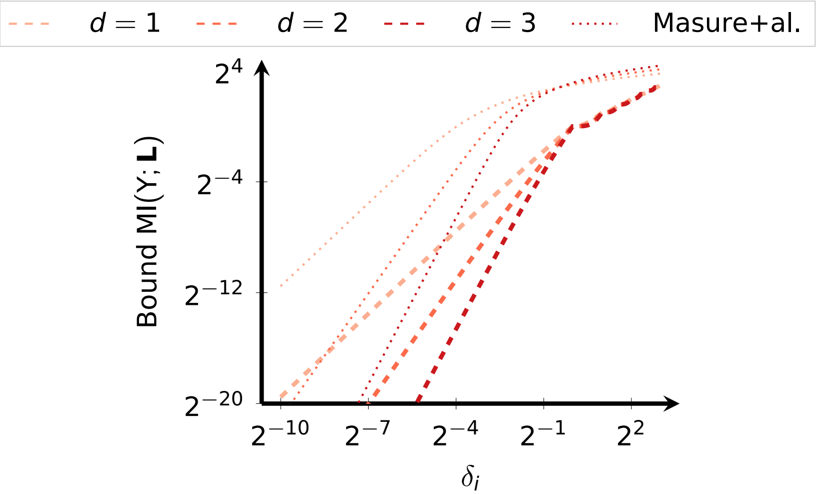
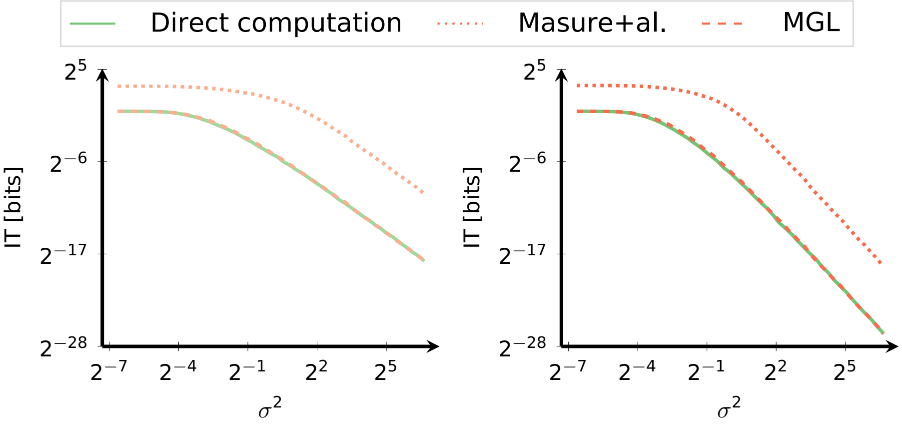


Figure: Illustration of the inequality for $M = 256$ (e.g., the AES S-box).

Practical Evaluation LSB

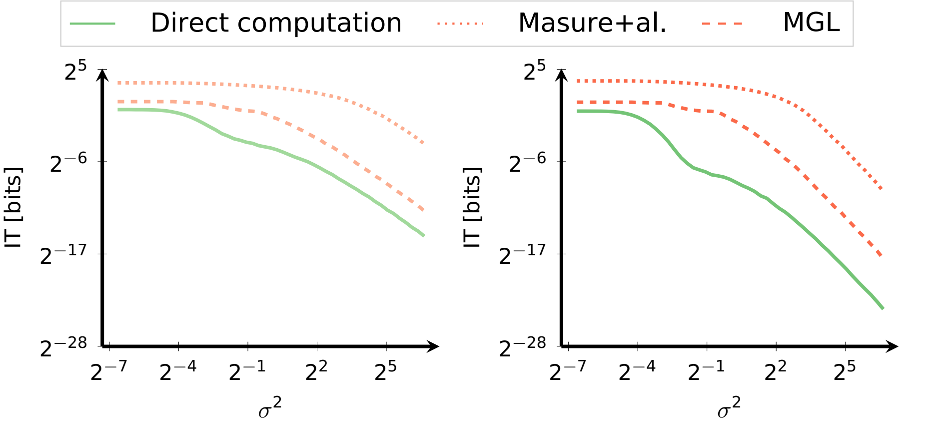


(a) lsb, $d = 1$.

(b) lsb, $d = 2$.

(c) MI in function of the Gaussian noise variance σ^2, for $n = 8$ bits.

Practical Evaluation HW

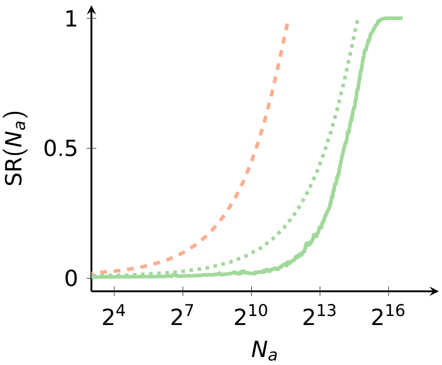


(a) HW, $d = 1$.

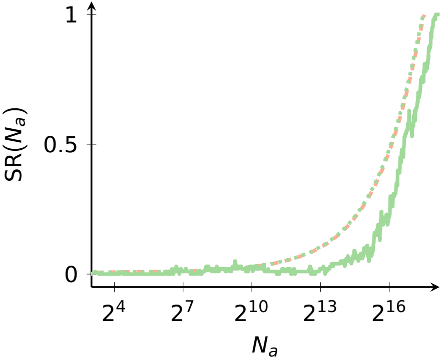
(b) HW, $d = 2$.

(c) MI in function of the Gaussian noise variance σ^2, for $n = 8$ bits.

Practical Evaluation $d = 1$



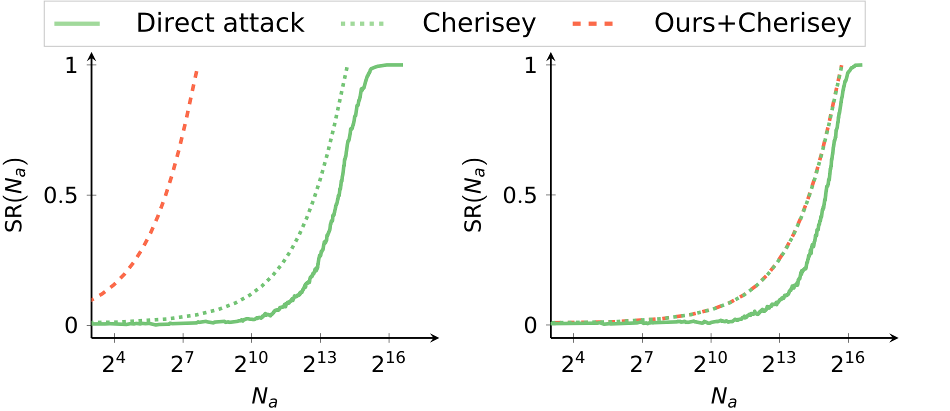
(a) HW.



(b) lsb.

Figure: Extending MI bounds to concrete security bounds for $\sigma^2 = 2^5, d = 1$.

Practical Evaluation $d = 2$



(a) HW. (b) Isb.

Figure: Extending MI bounds to concrete security bounds for $\sigma^2 = 2^2, d = 2$.

Conclusion & Perspectives

- We derived **optimal bounds** removing the field size from Duc+al. conjecture.
- Tighter bounds with mild assumptions ? n^{-d} for "generic leakages"
- Tightness for masked computations (e.g., multiplications) and not only encodings ?
- Extension to $M \neq 2^n$ especially for prime M ? We provide preliminary results using majorization arguments in the article.
- Other metrics (Rényi entropy/information, maximal leakage, etc) ?

Removing the Field Size Loss from Duc et al.'s Conjectured Bound for Masked Encodings

Thank you!

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Supplementary Material: Tighter MGL at the Bit Level ?

$$I(X_0 \star \dots X_d; Y_0 \dots Y_d) = \sum_{i=1}^{n} I((X_0 \star \dots X_d)_i; Y_0 \dots Y_d | (X_0 \star \dots X_d)_1^{i-1}) \quad (2)$$

$$\leqslant \sum_{i=1}^{n} I((X_0 \star \dots X_d)_i; Y_0 \dots Y_d | X_{0,1}^{i-1} \dots X_{d,1}^{i-1}) \quad (3)$$

$$\approx n^{-d} \varphi(\prod_j \varphi^{-1}(I(X_j; Y_j))) \quad (4)$$

Conjecture: We can still gain n^{-d} for "generic leakages"

Using Majorization

Theorem (Improved Bound for Generic Groups)

Let $P = \frac{1}{4}\prod_{i=0}^{d} C\,\mathrm{MI}(Y_i; L_i)$ *where* $C = 2/\log e$ *we have*

$$\mathrm{MI}(Y;\mathbf{L}) \leqslant \min\left(\log(1+M^2(4^{\frac{1}{M}}-1)P), (\frac{1}{M}+\sqrt{P})\log(1+M\sqrt{P})\right). \tag{5}$$