# Fault Attacks on UOV and Rainbow

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COSADE 2019

April 05, 2019

TECHNISCHE
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# Post-Quantum Cryptography

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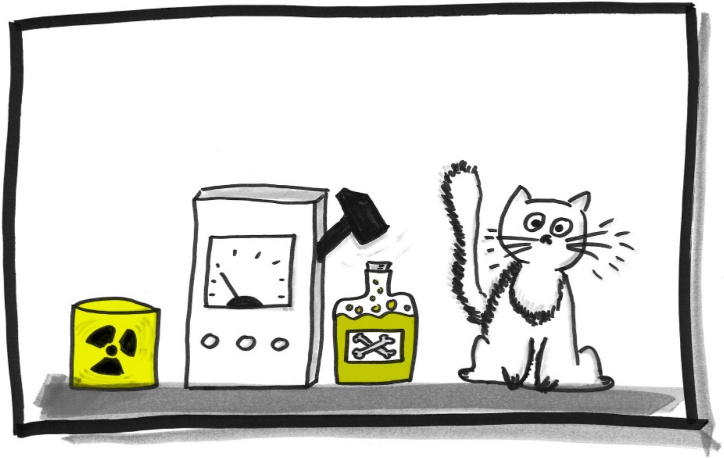
$\longrightarrow$ Classical (Public-Key) Cryptography

threatened by Quantum Computers

# Superposition



# 1994

Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer*

Peter W. Shor
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**Abstract**

A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time of at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, *e.g.,* the number of digits of the integer to be factored.

## Discrete Logarithm

Diffie-Hellman, DSA, El Gamal; ECC

## Factoring Problem

RSA

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OH ALICE... YOU'RE THE ONE FOR ME
BUT BOB... IN A QUANTUM WORLD HOW CAN WE BE SURE?
$\psi^+$ or $\psi^-$ ?


HOW'S YOUR QUANTUM COMPUTER PROTOTYPE COMING ALONG?
GREAT!
Dilbert.com DilbertCartoonist@gmail.com
THE PROJECT EXISTS IN A SIMULTANEOUS STATE OF BEING BOTH TOTALLY SUCCESSFUL AND NOT EVEN STARTED.
4-17-12 ©2012 Scott Adams, Inc./Dist. by Universal Uclick
CAN I OBSERVE IT?
THAT'S A TRICKY QUESTION.

# Candidates for Post-Quantum Cryptography

- … hash functions
- … isogenies
- … lattices
- … linear codes
- … multivariate equations

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# Multivariate cryptography

$$p^{(1)}(x_1, \ldots, x_n) = \sum_{i=1}^{n} \sum_{j=i}^{n} p_{ij}^{(1)} \cdot x_i x_j + \sum_{i=1}^{n} p_i^{(1)} \cdot x_i + p_0^{(1)}$$

$$\vdots$$

$$p^{(m)}(x_1, \ldots, x_n) = \sum_{i=1}^{n} \sum_{j=i}^{n} p_{ij}^{(m)} \cdot x_i x_j + \sum_{i=1}^{n} p_i^{(m)} \cdot x_i + p_0^{(m)}$$

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- central map $\mathcal{F} : \mathbb{F}^n \to \mathbb{F}^m$ (easily invertible)
- two invertible affine maps $\mathcal{T} : \mathbb{F}^m \to \mathbb{F}^m$ and $\mathcal{S} : \mathbb{F}^n \to \mathbb{F}^n$

- public key: $\mathcal{P} = \mathcal{T} \circ \mathcal{F} \circ \mathcal{S}$
- private key: $\mathcal{T}$, $\mathcal{F}$, and $\mathcal{S}$

# Multivariate cryptography

- message $d$
- hash value $\mathbf{w} = \mathcal{H}(d)$
- signature $\mathbf{z}$

**Signature Generation**

$$\mathbf{w} \in \mathbb{F}^m \xrightarrow{\mathcal{T}^{-1}} \mathbf{x} \in \mathbb{F}^m \xrightarrow{\mathcal{F}^{-1}} \mathbf{y} \in \mathbb{F}^n \xrightarrow{\mathcal{S}^{-1}} \mathbf{z} \in \mathbb{F}^n$$

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**Signature Generation**

$$\begin{tikzcd}
\mathbf{w} \in \mathbb{F}^m \arrow[r, "\mathcal{T}^{-1}"] & \mathbf{x} \in \mathbb{F}^m \arrow[r, "\mathcal{F}^{-1}"] & \mathbf{y} \in \mathbb{F}^n \arrow[r, "\mathcal{S}^{-1}"] & \mathbf{z} \in \mathbb{F}^n \arrow[lll, bend left, "\mathcal{P}"]
\end{tikzcd}$$

**Signature Verification**

# Fault Attacks on UOV and Rainbow

- targeting signature schemes UOV and Rainbow
- goal: reveal the affine maps $\mathcal{T}$ or $\mathcal{S}$
- decrease the complexity of a linear algebra attack by a fault attack

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- based on Hashimoto, Takagi, Sakurai: General fault attacks on multivariate public key cryptosystems, PQCrypto 2011

# Fault Attack on the Central Map 1/2

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$$\begin{aligned}\delta = h - h' &= \mathcal{P}'(z') - \mathcal{P}(z') \\ &= (\mathcal{T} \circ \mathcal{F}' \circ \mathcal{S})(z') - (\mathcal{T} \circ \mathcal{F} \circ \mathcal{S})(z') \\ &= (\mathcal{T} \circ \mathcal{F}' \circ \mathcal{S}(z')) - (\mathcal{T} \circ \mathcal{F} \circ \mathcal{S}(z')) \\ &= (T \circ (\mathcal{F}' - \mathcal{F}) \circ S)(z').\end{aligned}$$

# Fault Attack on the Central Map 2/2

- several rounds, at least $m - 1$
- deduce information about $T$, the linear part of $\mathcal{T}$
- apply the MinRank attack (with reduced complexity) to completely recover the affine map $\mathcal{T}$

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- 27 faults needed for current parameters

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- variables $x_i$, $i \in \{1, \ldots, n\}$ divided into $o$ oil and $v$ vinegar variables with $n = o + v$ and $v > o$
- vinegar variables randomly assigned during signature generation

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$\Rightarrow$ fixed-randomness attack

- $u_2$ number of fixed vinegar variables
- randomly chosen message $h^{(i)}, i \in \{1, \ldots, n - u_2 + 1\}$
- (faulty) signatures $z^{(i)}, i \in \{1, \ldots, n - u_2 + 1\}$

# Fault Attack on the Random Values 2/2

- (faulty) signatures allow simpler representation of $S$, the linear part of $\mathcal{S}$
- apply the MinRank attack (with reduced complexity) to completely recover the affine map $\mathcal{S}$
- success probability of $\geq 0.933$ for fields $\mathbb{F}_{16}$, $\mathbb{F}_{31}$, $\mathbb{F}_{256}$

# Countermeasures

- smaller fields are better protected than larger fields
  (e.g., $\mathbb{F}_{16}$ instead of $\mathbb{F}_{256}$)

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Attack on the Central Map

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Attack on the Random Values

- save and compare values

# Future Work

- Further analysis of countermeasures (more in the paper)
- Practical experiments
- Analyzing further attack vectors (FA and SCA)

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$\Rightarrow$ multivariate signature schemes inherently offer a good protection against fault attacks

# References


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All links have been opened on February 6, 2019, at 12:15 pm, for the last time

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(open PhD position ☺)