Collision-Correlation Attack against a First-Order Masking Scheme for MAC based on SHA-3

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Oberthur Technologies Crypto Group

COSADE 2014
April 14th and 15th, Paris

The Keccak Hash Function

- Bertoni, Daemen, Peeters, Van Assche
- SHA-3 winner
- Sponge construction

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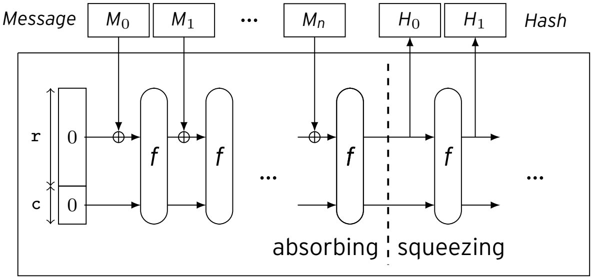
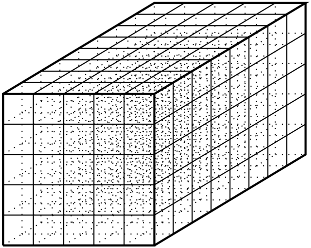


Security relies on the function f.

Keccak f function

State A: $5 \times 5 \times w$ bits.



for $i = 1$ **to** 24 **do**

$$A \leftarrow \iota_i \circ \chi \circ \pi \circ \rho \circ \theta(A)$$

end for

- θ xors columns
- ρ mix lanes
- π mix slices
- χ mix rows
- ι xor with cst

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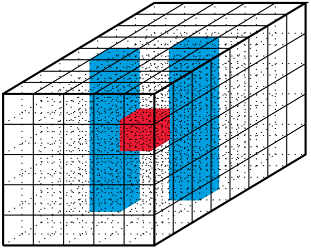

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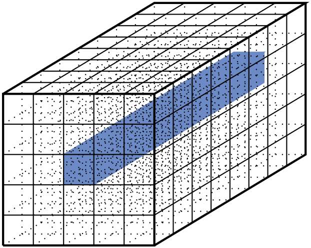

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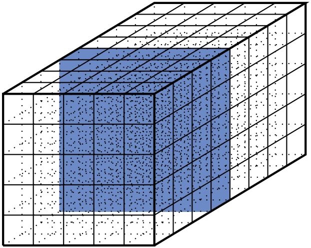

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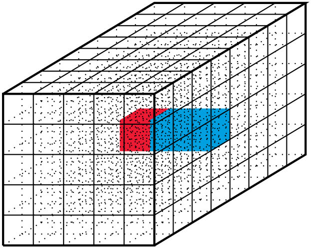

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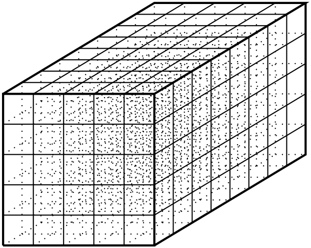

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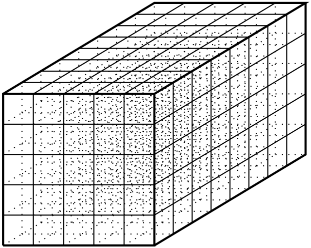

for $i = 1$ **to** 24 **do**

$A \leftarrow \lambda(A)$

$A \leftarrow \chi(A)$

$A \leftarrow A + K_i$

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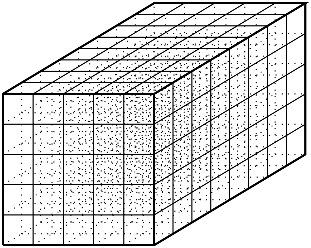

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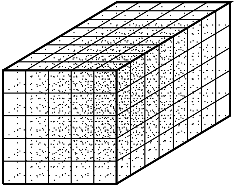
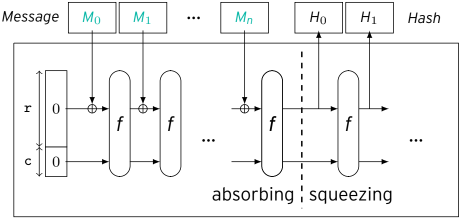
Practical Instantiation

$w = 64$ bits lane $\Rightarrow 1600$ bits state.

MAC based on Keccak

- Sponge construction $\Rightarrow$ no need for HMAC
- $\mathrm{MAC} = \lfloor H(K \| M) \rfloor_{\mathtt{p}}$





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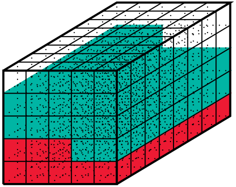

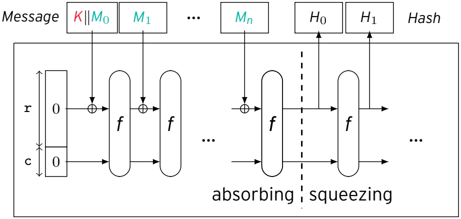

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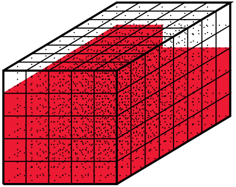

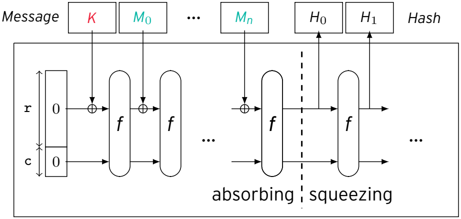

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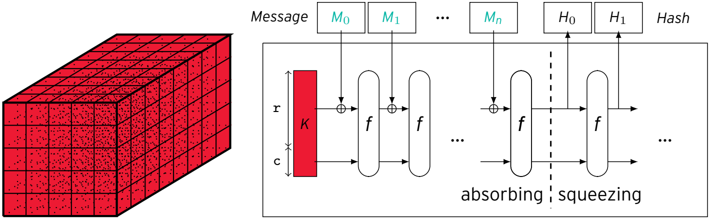

Always possible to consider a secret initial state.

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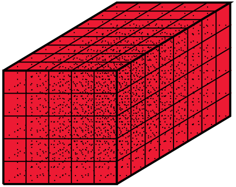

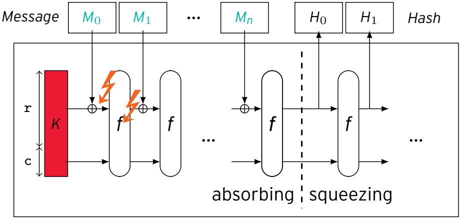

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DSCA on MAC based on Keccak

DSCA on keyed Keccak:

- Zohner, Kasper, Stöttinger, Huss, [DATE 2012]: brief analysis among other SHA-3 candidates.
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All prevented by first order masking.

Keccak First Order Masking Scheme

Already considered in SHA-3 submission:

- Boolean masking: $A = R \oplus S$
- Linear part: $\lambda(A) = \lambda(R) \oplus \lambda(S)$.
- Non-linear part: $a_x \leftarrow a_x + (a_{x+1} + 1) \cdot a_{x+2}$

$$r_x \leftarrow r_x + (r_{x+1} + 1) \cdot r_{x+2} + r_{x+1} \cdot s_{x+2}$$

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Masking Scheme Improvement (Keccak team, 2012)

- Precompute $Y = S \oplus \lambda(S)$,
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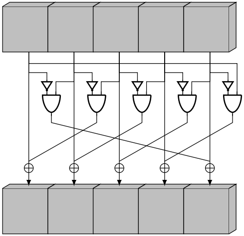
$\rightsquigarrow$ Possibility for collision-correlation.

Outline

1. Algebraic Collision Attack
2. Collision Detection
3. Experiments
4. Conclusion

Collisions in the χ function

χ function: $a_x \leftarrow a_x + (a_{x+1} + 1) \cdot a_{x+2}$ $\qquad (r_x = a_x + s_x)$.



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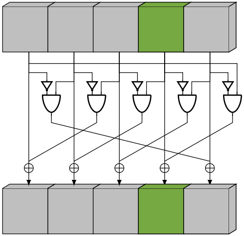

Bit collision:

$$a_x + s_x = a_x + (a_{x+1} + 1) \cdot a_{x+2} + s_x,$$

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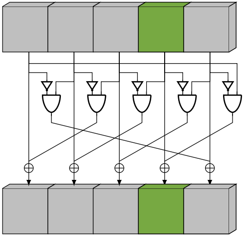

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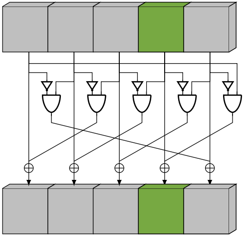

Bit collision first round:

$$a_x + s_x = a_x + (a_{x+1} + 1) \cdot a_{x+2} + s_x,$$
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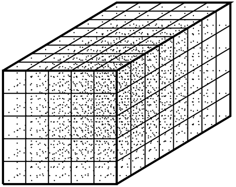
$$(\lambda(K \oplus M)_{x+1} + 1) \cdot \lambda(K \oplus M)_{x+2} = 0$$

First round: Each equation depend on 33 key bits.

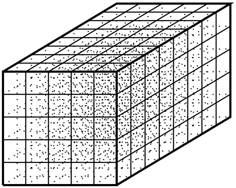
Building an Algebraic System



System of equations

Collisions in message

Building an Algebraic System



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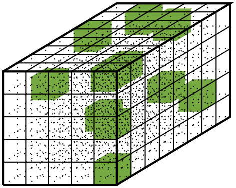

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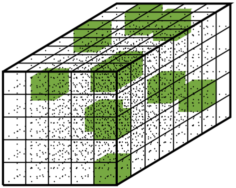

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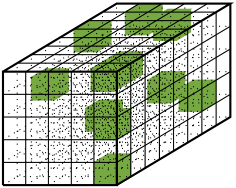

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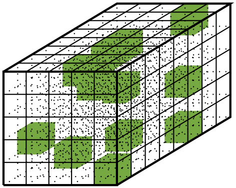

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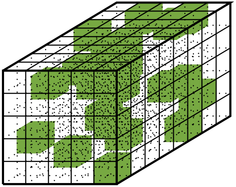

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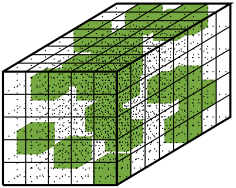

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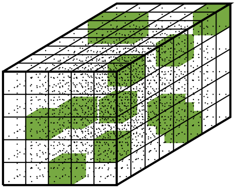

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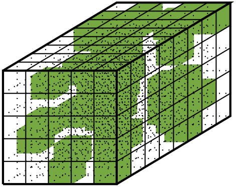

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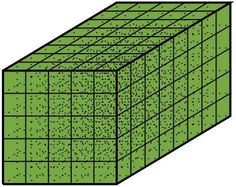

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Collisions in message

- 1600 variables and 1600+ equations...
- would take $\approx 2^{962}$ ops. for a random system

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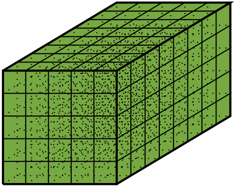

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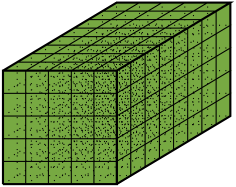

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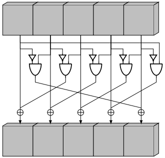
$\rightsquigarrow$ The system is heavily structured.

Considering Smaller Systems

Linearity of λ

By linearity, $\lambda(K \oplus M) = \lambda(K) \oplus \lambda(M) = K' \oplus M'$ where M' can be computed.

Linear change of variable $\Rightarrow$ reveals the structure: $5 \times w$ independent systems of 5 equations in 5 variables.



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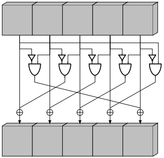

Easy to solve, even with exh. search.

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Attack Roadmap

1. Detect collisions from n different messages.
2. Build $5 \times w$ small systems $\mathcal{F}_{y,z}$.
3. Solve each system $\mathcal{F}_{y,z} \Rightarrow \mathcal{V}_{y,z}$.
4. Build all candidates K' from $\mathcal{V}_{y,z}$.
5. Compute $K = \lambda^{-1}(K')$ for all K'. Exhaustively search the correct key.

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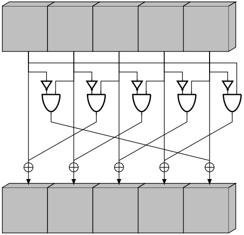
Collision Detection by Correlation



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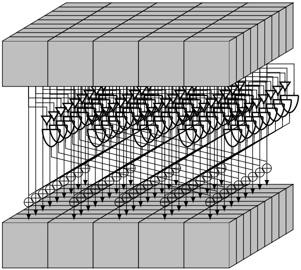

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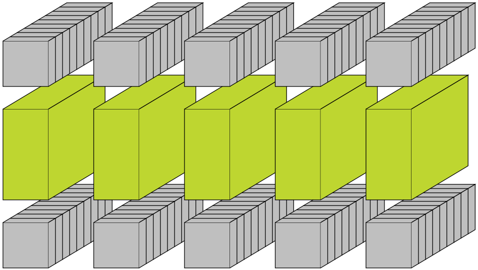
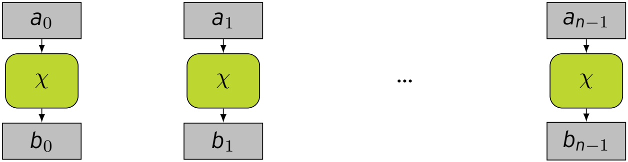


- Difficult to detect a collision on a bit;
- χ is usually processed by machine word (ℓ bits);
- Consider each word independently.

Collision Detection by Correlation (cont'd)



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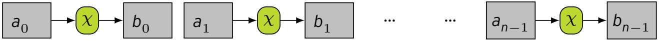

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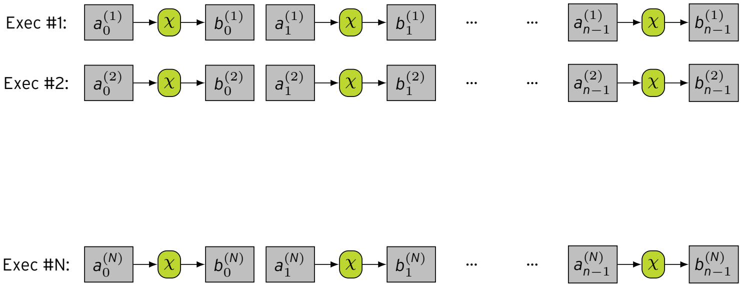

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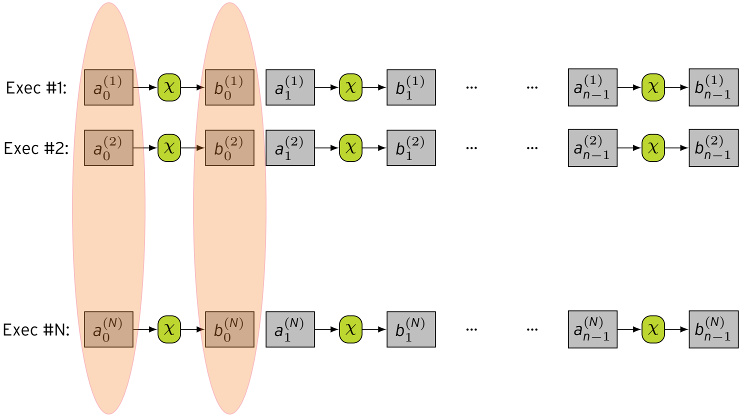

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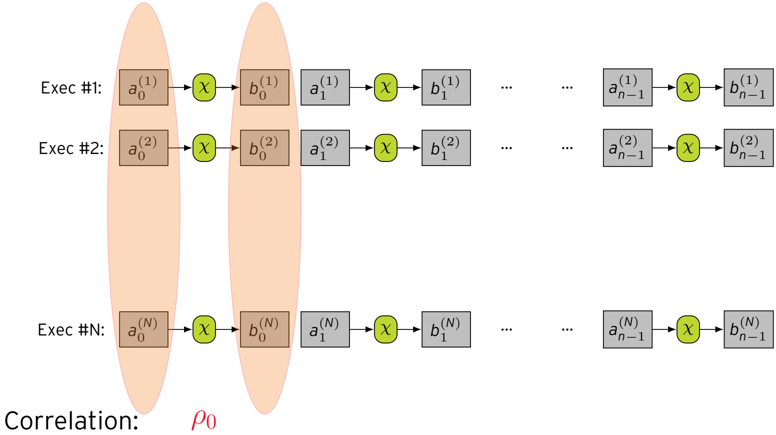

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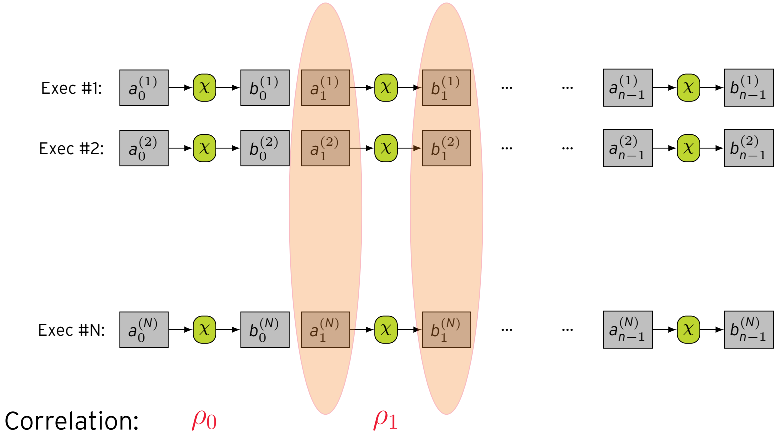

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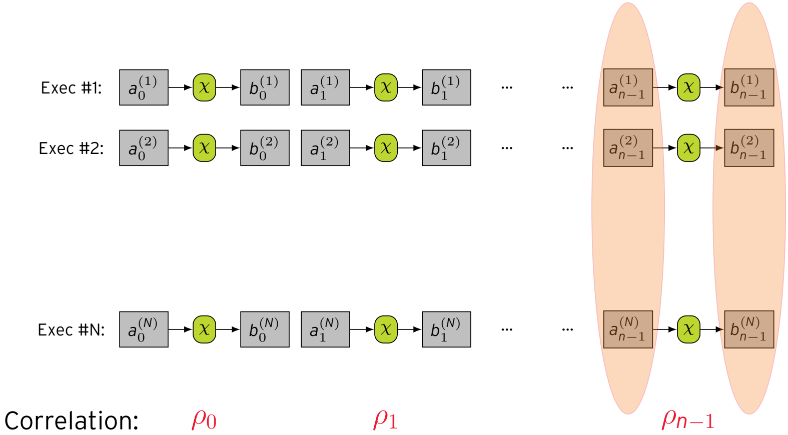
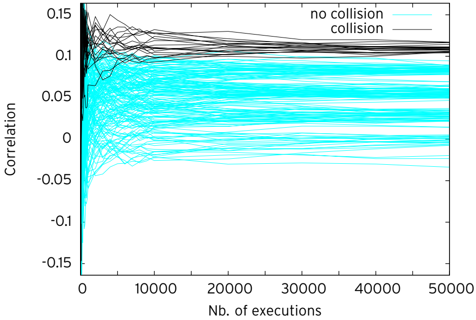


Recognize a Collision (HW leakage model)

Correlations for one message $\ell = 8$, noise $\sigma = 4$.



- Correlations appear by packs
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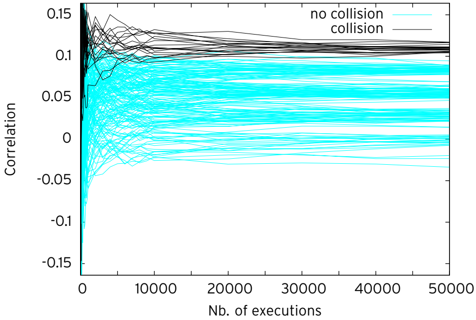

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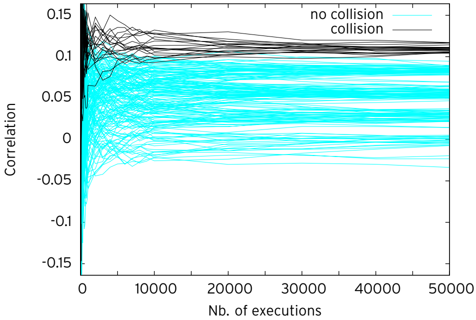

- Correlations appear by packs $\Rightarrow$ related to HW
- More top correlations $\Rightarrow$ High collision probability

Required Number of Messages

Attack Behavior depending on #messages, $\ell = 8$ (average):

#messages	0	1	20	50	60
#collisions	0	20.4	177.3	199.0	199.7
#equations	0	0.5	7.4	12.2	13.1
#candidates	2^{1600}	2^{1558}	2^{589}	2^{78}	2^{38}
#messages	70	80	90	140	170
#collisions	199.8	199.9	200	200	200
#equations	13.6	14.0	14.3	14.9	15.0
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- Only 70 *different* messages are needed;
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Conclusion

Summary

- Combining collision correlation and algebraic attack;
- High collision probability helps our attack;
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- High collision probability helps our attack;
- Efficient detection in random leakage model.

- Other collision detection techniques ?
- Find an *efficient* masking scheme ?

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Each input bit of χ has a probability $\frac{3}{4}$ to collide with its output bit.

$$a_x = a_x + (a_{x+1} + 1) \cdot a_{x+2} \,.$$

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Collisions in Keccak

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Probability of (at least one) collision in a message:

bit-size ℓ	8	16	32
prob. collision	≈ 1	0.635	0.005

High probability of collision $\Rightarrow$ no threshold for detection.

Considering Other Leakage Models

Experimental Framework

Leakage function L, let $a = \sum_{i=0}^{\ell-1} a_i$:

- *quad* leakage: $L(a) = f_{\text{quad}}(a_0, \ldots, a_{\ell-1})$, $\deg f_{\text{quad}} = 2$;
- *full*: leakage: $L(a) = f_{\text{full}}(a_0, \ldots, a_{\ell-1})$, $\deg f_{\text{full}} = \ell$.

Noise level: keep constant Signal to Noise Ratio (SNR).

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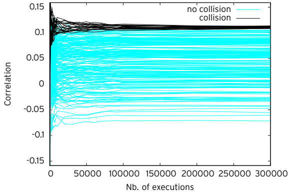
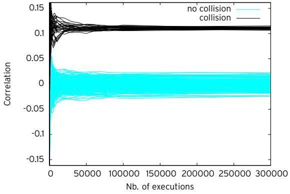
Correlations for one message $\ell = 8$, SNR=0.125.



quad.



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Comparison to 2O DSCA

2O-DSCA on Keccak

- Correlation between χ input/output ℓ bits words;
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Comparison to 2O-DSCA for $\ell = 8$, SNR=0.125:

	HW	*quad*	*full*
This attack	$315\,000 \times 70$	$200\,000 \times 70$	$5\,000 \times 70$
2O-DSCA	$600\,000$	$> 1\,500\,000$	$> 1\,500\,000$